**Excitations**

So we’re going to start with:



and of course the momenta are presumed restricted to close to the Fermi surface. I guess we’ll say |ξk(0)| < ωH, whatever ωH might be in this case, in analogy with the BCS Hamiltonian. And we’re going to adapt the manipulations we made on the BCS Hamiltonian to this one. We’ll assume the He’s pair in equal/opposite momenta states. In the BCS case, we presumed the electrons paired up in a spin-singlet state, and here we won’t make any presumptions on the spins. As before we take the Cooper pair destruction / creation operators to be non-fluctuaty.



So let’s define:



and then we can write:



(remember that in the BCS model we’re in an ensemble of many-body states with different particle numbers, and so these expectations don’t have to be zero) Filling this in,



where in the last line we ignore the (δg)2 term as being small – that’s the mean field approximation. Filling the guys back in we have:



We can neglect the constant, and fill gkσσ´ back in,



Now let’s *define*:



Then we can write H as:



So we have:



Reassuringly, our H is hermitian.

**Solving for the GF’s**

We can try to find a unitary transformation that diagonalizes our bilinear Hamiltonian. The spins make it complicated. But could probably adapt the method used in the Superconductor/Excitations/MFT appendix. Guess I’m going to do the GF equation of motion thing instead. That’s complicated too.

Well let’s define a self-consistent set of GF’s. There’s a lot of them now because we have to consider all the different spin configurations. We’re going to consider a very general set of GF’s. If you take a look at the Symmetry Considerations file, you can see that unless we know what Δσσ´(k) is doing, we can’t assume basically anything about the symmetry of the GF’s, except that they conserve momentum. Otherwise, all bets are off, especially apropos spin symmetry. And this is necessary, since we’re ultimately looking for a more general spin arrangement than we found for BCS superconductors. What’s more, I think, since we’re trying to describe the liquid helow below Tc, this is where we can get spontaneous symmetry breaking, as we see for the Ferromagnetic phase transition in iron (see Stat Mech folder). So as the name implies, we can’t expect the correlation functions to have this symmetry either. So here we are.



Well, we need equations of motion. So let’s differentiate cps w/r to time.



where,



Need those commutators,



and also have (they’re not zero! Fermions!):



Ok maybe zero. Oh yeah and,



Plugging these results back in,



Using that anti-symmetric property of Δ from the Symmetry considerations file,



we can say,



And taking the dagger of this equation (well, convert to real time first, and then dagger it, or take dagger of the ∂c/∂τ = [K,c] equation and can see it amounts to same thing), we have all together:



So now putting this in G’s spot.



Simplifying some…and note in the δ function terms, we can set τ = 0 in cpσ(H), and then combine those terms…the anticommutator will give us 1. And then the two θ guys will give us the Time ordering operator back again. So,



Now filling in the derivative,



As for F,



Simplifying some…and note in the δ function terms, we can set τ = 0 in cpσ(H), and then combine those terms…the anticommutator will give us 0 this time. And then the two θ guys will give us the Time ordering operator back again. So,



And now let’s do the last one,



And now plug in the equations of motion, and use that anti-symmetric Δ property:



So altogether, our equations are:



So these form a self-consistent set, and we can solve them to find the energy levels an such. Let’s take the Fourier transform, ∫0βdτ eiω\_nτ…



For the middle one, we have:



Now going to change variables τ → τ-β, and use: GC\*(τ-β) = -εGC\*(τ),



And for the last one, we have:



And so altogether,



The first and last are independent. Maybe I’ll just solve for F† in the last,



and plug into the first,



Well now we have just a 2×2 matrix equation to solve! We can write an operator GF and operator ‘gap-function’, where |σ> = (1 0) for up states and (0 1) for down states.



and in terms of this, we can formally write and solve our equation:



So need to find the eigenvalues of this ‘gap-function’ matrix. Our matrix is:



Going through the motions, we need to solve:



Gonna call these eigenvalues λ± = Δ(2)±(p). So we have:



Now let |ψ+(p)> be the eigenvector of our gap-function matrix associated with eigenvalue Δ(2)+, and |ψ-(p)> be the one associated with eigenvalue Δ(2)-. We can then write the operator gap function as:



So then we can write our solution as:



And in terms of components,



As for the other guys, we have:



and



So altogether,



The poles of the retarded GF (iωn → ω + i0+) determine the energy levels, and these are given by:



Like with the BCS case, we can see that a gap has opened up in the free particle spectrum, around the Fermi surface.

**The Gap Equation**

Now to get the behavior of the gap function, we need to solve for it via our self-consistent equations. Recalling,



We can say,



So we need to solve,



We can do the Matsubara sum over the frequencies at least,



And then we use the identity,



So,



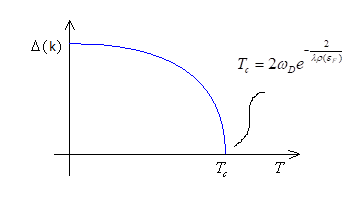
And since:



we can say,



There are many possible solutions to this non-linear equation. To make further progress we kind of need to make some assumptions. First, recall that when we studied superconductors, the BCS gap function Δ(k) looked like this:



The BCS gap function had a critical temperature below which electrons would start to aggregate into Cooper pairs which are singlet spin states. Now we have four gap functions:



, each of which we can imagine displays some similar sort of behavior as a function of T. Each of these gap functions may have their own Tc. We will also find a more interesting behavior as a function of k. The BCS gap function was a constant over that 2ωD interval straddling the Fermi surface. But here we’ll find the gap function can depend in a non-trivial way on k (well, its angle, not its magnitude), around that 2ωH interval about the Fermi surface. And moreover, there are more than one single solution these gap functions may take, unlike in the BCS case where there was only one solution to Δ(k). Recall that F†σσ´ is the GF which creates a pair of 3He atoms in the σσ´ spin state. And so Δσσ´(k) ~ Fσσ´ can be thought of sort of ‘amplitude’ for the electrons to pair into a σσ´ state. And we could write the general predilection for two 3He atoms to pair up in a certain spin configuration as:



Another possibility is to define an amplitude for the electrons to pair up in a spin singlet state, or any of the three spin triplet states.



And then we could write the general predilection of two 3He atoms to pair into one of these states as:



This suggests the identification/definition:



Turns out that for the most part, 3He will pair up in either a singlet or triplet state, but not both. So we look for just singlet or triplet solutions. We’ll continue this line of thought in the next file.

**Appendix**

So,



and,



Are the eigenvalues of Δ†Δ the same as those of ΔΔ†? Well at least for 2×2 matrices they are, since these two matrices have the same trace and determinant → λ1 + λ2 is same, and λ1λ2 is same too. But eigenvectors may be quite different, as the Pauli matrices all have same eigenvalues too. So I don’t think the eigenvectors will be identical too, as then we’d have: Δ†Δ = ΔΔ†. And I don’t think they’re necessarily identical.